Macroscopic modeling and simulation of crowd dynamics

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Outline of the talk

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Mathematical modeling of pedestrian motion: frameworks

Microscopic

- individual agents
- ODEs system
- many parameters
- low and high densities
- comp. cost $\sim$ ped. number.

Macroscopic

- continuous fluid
- PDEs
- few parameters
- very high densities
- analytical theory
- comp. cost $\sim$ domain size

Macroscopic models

- Pedestrians as "thinking fluid"[1]
- Averaged quantities:
  - $\rho(t, \mathbf{x})$ pedestrians density
  - $\vec{v}(t, \mathbf{x})$ mean velocity

Mass conservation

$$\begin{cases} \partial_t \rho + \mathrm{div}_{\mathbf{x}}(\rho \vec{v}) = 0 \\ \rho(0, \mathbf{x}) = \rho_0(\mathbf{x}) \end{cases}$$

for $\mathbf{x} \in \Omega \subset \mathbb{R}^2$, $t > 0$

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Two classes

- **1st order models:** velocity given by a phenomenological *speed-density relation* $\vec{v} = V(\rho)\vec{\nu}$
- **2nd order models:** velocity given by a *momentum balance equation*

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- Density must stay non-negative and bounded: $0 \leq \rho(t, \mathbf{x}) \leq \rho_{\max}$
- Different from fluid dynamics:
  - preferred direction
  - no conservation of momentum / energy
  - $n \ll 6 \cdot 10^{23}$

[1] R.L. Hughes, Transp. Res. B, 2002

Continuum hypothesis

$n \ll 6 \cdot 10^{23}$ but ...

Brown University, Main Green, 08.21.2017

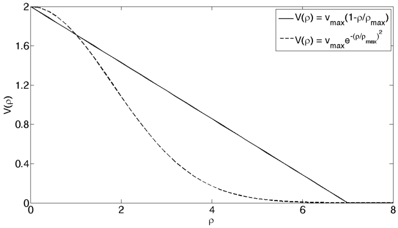
Speed-density relation

Speed function $V(\rho)$:

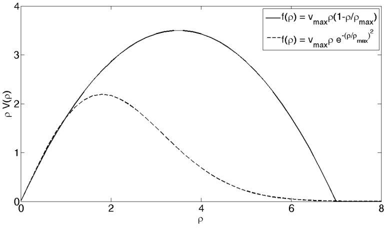
- decreasing function wrt density
- $V(0) = v_{\max}$ **free flow**

  $V(\rho_{\max}) \simeq 0$ **congestion**

Examples:



speed $V(\rho)$



flux $\rho V(\rho)$

Desired direction of motion $\vec{\mu}$

Pedestrians:

- seek the shortest route to destination
- try to avoid high density regions

$$\vec{\nu} = -\frac{\nabla_{\mathbf{x}}\phi}{|\nabla_{\mathbf{x}}\phi|}$$

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The potential $\phi : \Omega \to \mathbb{R}$ is given by the Eikonal equation

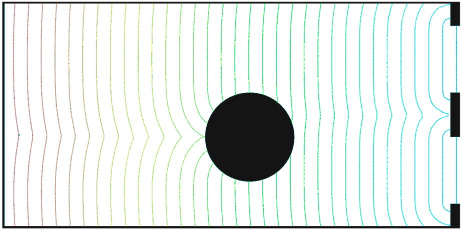
$$\begin{cases} |\nabla_{\mathbf{x}}\phi| = C(t, \mathbf{x}, \rho) & \text{in } \Omega \\ \phi(t, \mathbf{x}) = 0 & \text{for } \mathbf{x} \in \Gamma_{outflow} \end{cases}$$

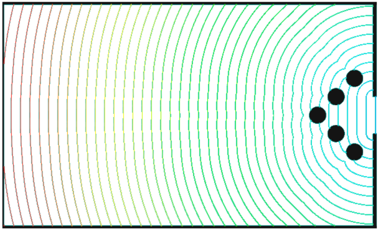
where $C = C(t, \mathbf{x}, \rho) \geq 0$ is the *running cost*

$\Longrightarrow$ the solution $\phi(t, \mathbf{x})$ represents the weighted distance of the position $\mathbf{x}$ from the target $\Gamma_{outflow}$

Eikonal equation: level set curves for $|\nabla_{\mathbf{x}}\phi| = 1$

In an empty space: potential is proportional to distance to destination





The fastest route ...

... needs not to be the shortest!

First order models

- Hughes' model[1]

$$\vec{\nu} = -\frac{\nabla_{\mathbf{x}}\phi}{|\nabla_{\mathbf{x}}\phi|} \quad \text{s.t.} \quad |\nabla_{\mathbf{x}}\phi| = \frac{1}{V(\rho)}$$

  - minimize travel time avoiding high densities
  - CRITICISM: instantaneous global information on entire domain

[1] R.L. Hughes, Transp. Res. B, 2002
[2] Y. Xia, S.C. Wong and C.-W. Shu, Physical Review E, 2009
[3] R.M. Colombo, Garavello and M. Lécureux-Mercier, M3AS, 2012

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- Dynamic model with memory effect[2]

$$\vec{\nu} = -\frac{\nabla_{\mathbf{x}}(\phi+\omega D)}{|\nabla_{\mathbf{x}}(\phi+\omega D)|} \quad \text{s.t.} \quad |\nabla_{\mathbf{x}}\phi| = \frac{1}{v_{\max}}, \; D(\rho) = \frac{1}{v(\rho)} + \beta\rho^2 \quad \textit{discomfort}$$

  - minimize travel time based on knowledge of the walking domain
  - temper the behavior locally to avoid high densities

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- Non-local flow:[3]

$$\vec{v} = V(\rho)\left(\vec{\nu} - \varepsilon\frac{\nabla(\rho * \eta)}{\sqrt{1 + |\nabla(\rho * \eta)|^2}}\right) \quad \text{with} \quad \vec{\nu} = -\frac{\nabla_{\mathbf{x}}\phi}{|\nabla_{\mathbf{x}}\phi|} \quad \text{s.t.} \; |\nabla_{\mathbf{x}}\phi| = 1$$

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Second order model

Momentum balance equation[4][5]

$$\partial_t(\rho\vec{v}) + \mathrm{div}_{\mathbf{x}}(\rho\vec{v}\otimes\vec{v}) + \nabla_{\mathbf{x}}P(\rho) = \rho\frac{V(\rho)\vec{\nu} - \vec{v}}{\tau}$$

where

- $V(\rho) = v_{\max} e^{-\alpha\left(\frac{\rho}{\rho_{\max}}\right)^2}$
- $|\nabla_{\mathbf{x}}\phi| = 1/V(\rho)$
- $P(\rho) = p_0\rho^\gamma$, $p_0 > 0$, $\gamma > 1$ internal pressure
- τ response time

[4] Payne-Whitham, 1971

[5] Y.Q. Jiang, P. Zhang, S.C. Wong and R.X. Liu, Physica A, 2010

Question

Can macroscopic models reproduce characteristic features of crowd behavior?

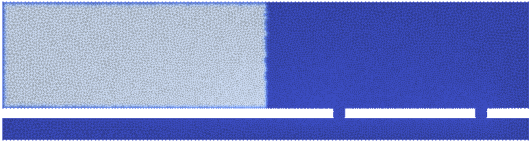
Outline of the talk

Numerical schemes used

- **Space meshes:** unstructured triangular / cartesian
- **Eikonal equation:** linear, finite element solver[6] / fast-sweeping
- **First order models:** Lax-Friedrichs
- **Second order models:** explicit time integration with advection-reaction splitting (HLL scheme)
- **Non-local models:** dimensional splitting Lax-Friedrichs

[6] [Bornemann-Rasch, 2006]

Corridor evacuation with two exits



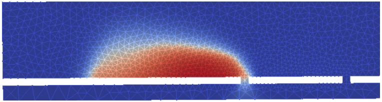
Configuration at $t = 0$

Parameters choice:

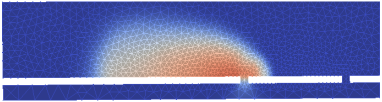
$\rho_0 = 3\text{ped}/m^2$ initial density
$\rho_{\max} = 10\text{ped}/m^2$ maximal density
$v_{\max} = 2m/s$ desired speed
$\tau = 0.61s$ relaxation time
$p_0 = 0.005\text{ped}^{1-\gamma}m^{2+\gamma}/s^2$ pressure coefficient
$\gamma = 2$ adiabatic exponent
$\alpha = 7.5$ density-speed coefficient
$\varepsilon = 0.8$ correction coefficient
$\eta = [1-(x/r)^2]^3[1-(y/r)^2]^3$ convolution kernel, with $r = 15m$

Corridor evacuation with two exits

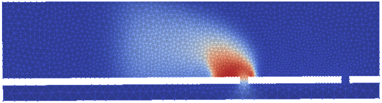
$t = 20s$



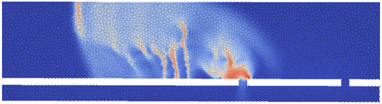
$|\nabla_{\mathbf{x}}\phi| = 1$



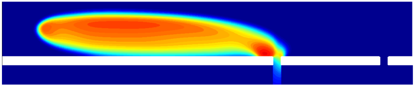
$\nabla_{\mathbf{x}}(\phi + \omega D)$



$|\nabla_{\mathbf{x}}\phi| = 1/v(\rho)$



second order

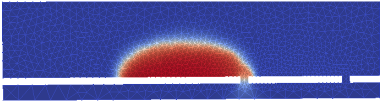


non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]

Corridor evacuation with two exits

$t = 40s$



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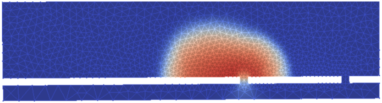

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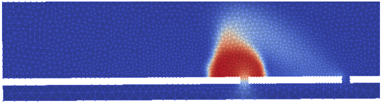

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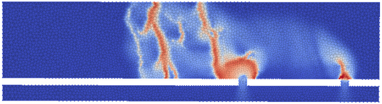
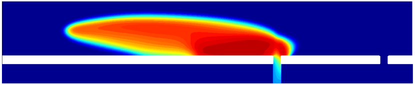


second order



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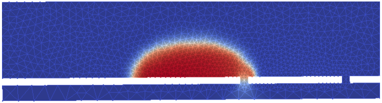
Corridor evacuation with two exits

$t = 60s$



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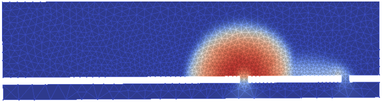

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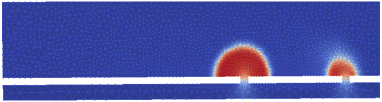

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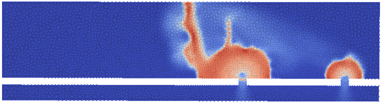
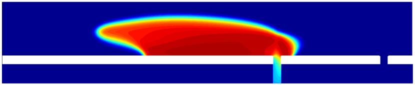


second order

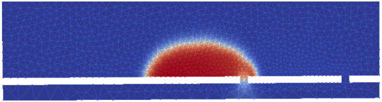


non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]

Corridor evacuation with two exits

$$t = 80s$$



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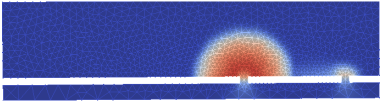

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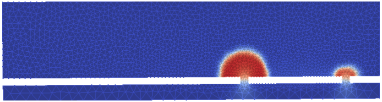

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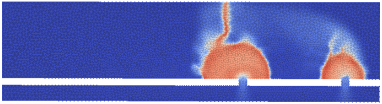

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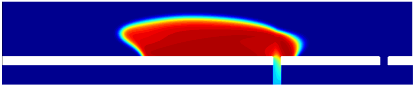

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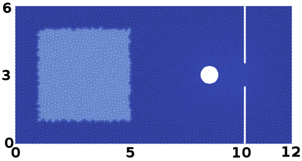
[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]

Room evacuation with obstacle



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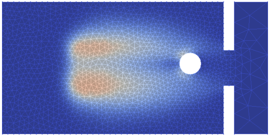
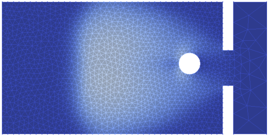
Room evacuation with obstacle

$t = 2s$



$|\nabla_{\mathbf{x}}\phi| = 1$



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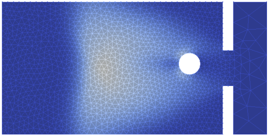

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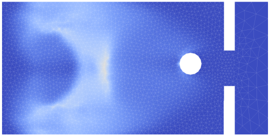
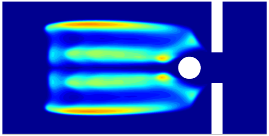


second order

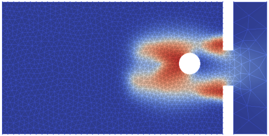


non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]

Room evacuation with obstacle

$t = 5s$



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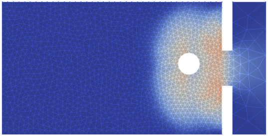

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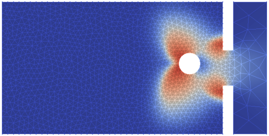

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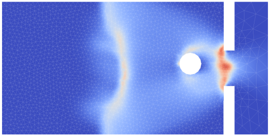
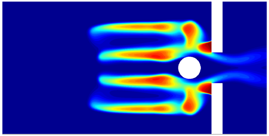


second order

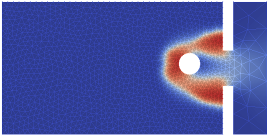


non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]

Room evacuation with obstacle

$t = 8s$



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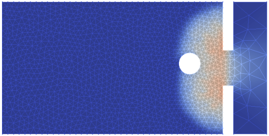

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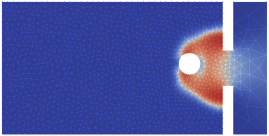

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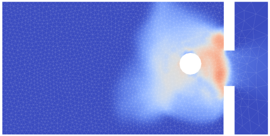
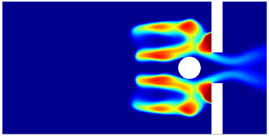


second order

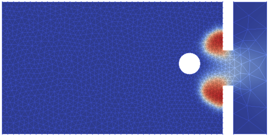


non-local

[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]

Room evacuation with obstacle

$t = 11s$



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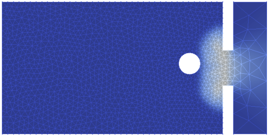
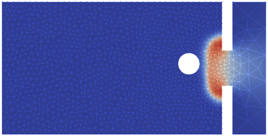
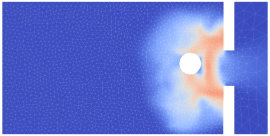


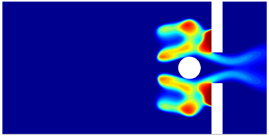
$\nabla_{\mathbf{x}}(\phi + \omega D)$



$|\nabla_{\mathbf{x}}\phi| = 1/v(\rho)$



second order

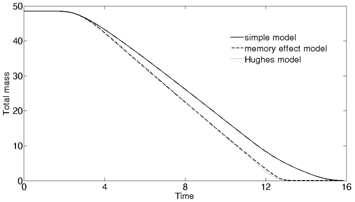


non-local

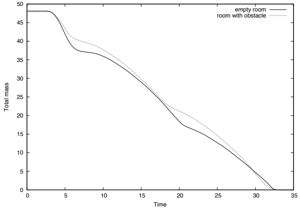
[Twarogowska-Duvigneau-Goatin, Mimault-Goatin]

Effect of the obstacle on the outflow

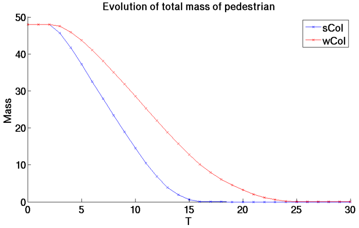
Time evolution of the total mass of pedestrians inside the room



first order



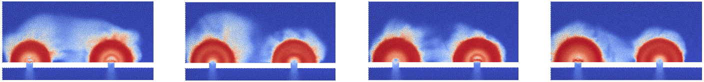
second order



non-local

$$M(t) = \int_{\Omega} \rho(t,x)dx$$

Second order model: stop-and-go waves



t=90 t=100 t=110 t=120

$$P(\rho) = 0.005\rho^2, \qquad v_{\max} = 2, \qquad \rho_{\max} = 7$$

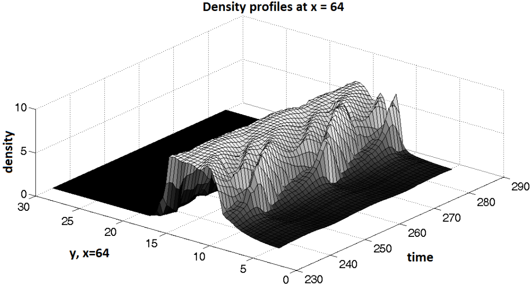
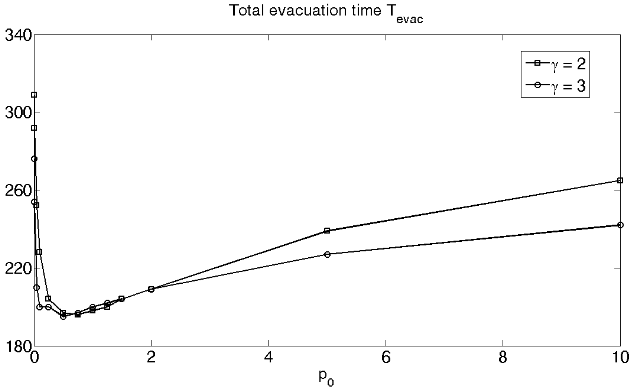


Fig. Time evolution of density profile at $x = 64$ (left exit)

Second order model: dependence on p_0

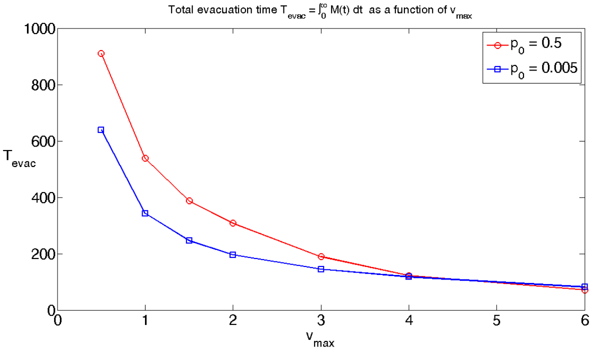
$P(\rho) = p_0 \rho^\gamma$: total evacuation time optimal for $p_0 \sim 0.5$



with $v_{\max} = 2m/s$, $\rho_{\max} = 7\text{ped}/m^2$

Second order model: dependence on $v_{\max}$

Total evacuation time

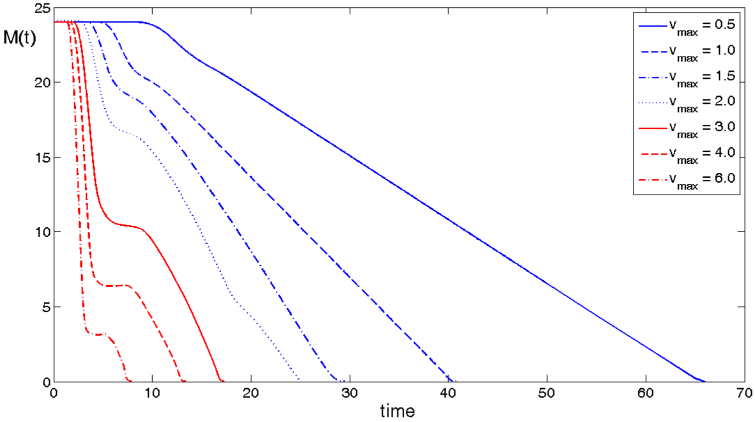


Social force models[7] show a minimum for $v_{\max} \simeq 1.4\ m/s$
$\Longrightarrow$ **faster-is-slower effect**[8]
Accounting for inter-pedestrian friction?

[7] D. Helbing, I. Farkas and T. Vicsek, Nature, 2000
[8] D.R. Parisi and C.O. Dorso, Physica A, 2007

Second order model: dependence on $v_{\max}$

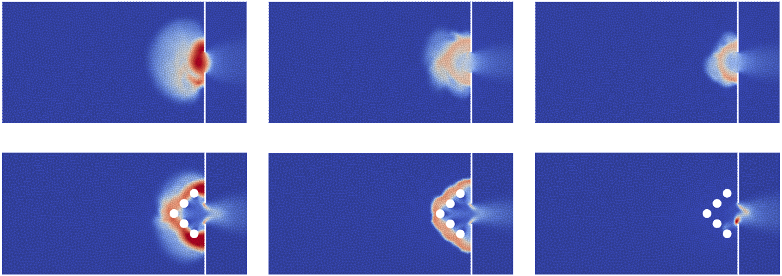
Total mass evolution



with $\rho_{\max} = 7\text{ped}/m^2$, $\gamma = 2$, $p_0 = 0.005$

Evacuation optimization: Braess' paradox[9] ?

Problem: **clogging** at exit

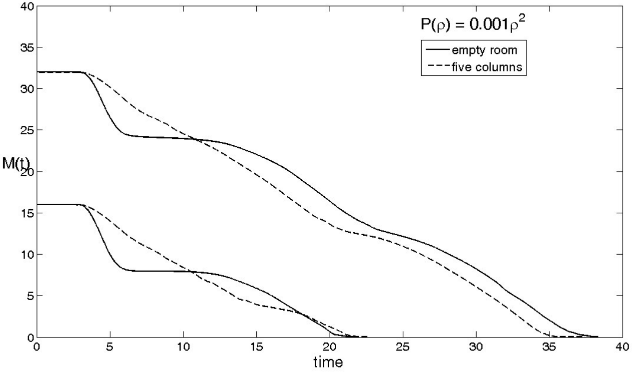


Can obstacles reduce the evacuation time?

[9] Braess, D. *Über ein Paradoxon aus der Verkehrsplanung*, Unternehmensforschung, 12, pp. 258-268 (1968)

Evacuation optimization: Braess' paradox?

Time evolution of the total mass of pedestrians inside the room



Non-local model: lane formation[10]

Two groups of pedestrians moving in opposite directions

$$\begin{cases} \partial_t U^1 + div\left(c_1 U^1(1-U^1)\left(\left(1-\epsilon_1 \frac{U^1*\mu}{\sqrt{1+\|U^1*\mu\|^2}}\right)\vec{v}^1(x,y) - \epsilon_2 \frac{\nabla U^2*\mu}{\sqrt{1+\|\nabla U^2*\mu\|^2}}\right)\right) = 0, \\ \partial_t U^2 + div\left(c_2 U^2(1-U^2)\left(\left(1-\epsilon_1 \frac{U^2*\mu}{\sqrt{1+\|U^2*\mu\|^2}}\right)\vec{v}^2(x,y) - \epsilon_2 \frac{\nabla U^1*\mu}{\sqrt{1+\|\nabla U^1*\mu\|^2}}\right)\right) = 0. \end{cases}$$

where

$$c_1 = c_2 = 4 \qquad \text{crowding factor}$$
$$\epsilon_1 = 0.3, \quad \epsilon_2 = 0.7,$$

can be derived as mean-field and hydrodynamic limit of microscopic model [Göttlich-Klar-Tiwari, JEM 2015]

[10] R.M. Colombo and M. Mercier, Acta Mathematica Scientia, 2011

Lane formation in bidirectional flows

[Aggarwal-Colombo-Goatin, SINUM 2015; Aggarwal-Goatin, BBMS 2016]

Lane formation in crossing flows

[Aggarwal-Colombo-Goatin, SINUM 2015; Aggarwal-Goatin, BBMS 2016]

Outline of the talk

The 1D case: statement of the problem

We consider the initial-boundary value problem

$$\begin{aligned} &\rho_t - \left(\rho(1-\rho)\frac{\phi_x}{|\phi_x|}\right)_x = 0 \\ &|\phi_x| = c(\rho) \end{aligned} \qquad x \in \Omega =]-1,1[, \; t > 0$$

with initial density $\rho(0,\cdot) = \rho_0 \in \mathrm{BV}(]0,1[)$
and *absorbing* boundary conditions

$$\begin{aligned} &\rho(t,-1) = \rho(t,1) = 0 \qquad \text{(weak sense)} \\ &\phi(t,-1) = \phi(t,1) = 0 \end{aligned}$$

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General cost function c: $[0,1[\ \to [1,+\infty[$ smooth s.t. $c(0) = 1$ and $c'(\rho) \geq 0$
(e.g. $c(\rho) = 1/v(\rho)$)

The 1D case: statement of the problem

The problem can be rewritten as

$$\rho_t - \Big(\mathrm{sgn}(x - \xi(t))\ f(\rho)\Big)_x = 0$$

where the *turning point* is given by

$$\int_{-1}^{\xi(t)} c\left(\rho(t,y)\right)\ dy = \int_{\xi(t)}^{1} c\left(\rho(t,y)\right)\ dy$$

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$\longrightarrow$ the discontinuity point $\xi = \xi(t)$ is not fixed *a priori*, but depends *non-locally* on ρ

The 1D case: available results

- **existence and uniqueness** of Kruzkov's solutions for an elliptic regularization of the eikonal equation and $c = 1/v$
  [DiFrancesco-Markowich-Pietschmann-Wolfram, JDE 2011]
- **Riemann solver** at the turning point for $c = 1/v$
  [Amadori-DiFrancesco, Acta Math. Sci. B 2012]
- **entropy condition and maximum principle**
  [ElKhatib-Goatin-Rosini, ZAMP 2012]
- **wave-front tracking algorithm** and convergence of finite volume schemes
  [Goatin-Mimault, SISC 2013]
- **existence** for data with small L^∞ and TV norms and $c = 1/v$
  [Amadori-Goatin-Rosini, JMAA 2013]
- **local** version
  [Carrillo-Martin-Wolfram, M3AS 2016]
- extension to **graphs**
  [Camilli-Festa-Tozza, NHM 2017]

The 1D case: entropy condition

Definition: entropy weak solution (ElKhatib-Goatin-Rosini, 2012)

$\rho \in \mathbf{C^0}\left(\mathbb{R}^+;\mathbf{L^1}(\Omega)\right) \cap \mathrm{BV}\left(\mathbb{R}^+ \times \Omega;[0,1]\right)$ s.t. for all $k \in [0,1]$ and $\psi \in \mathbf{C_c^\infty}(\mathbb{R}\times\Omega;\mathbb{R}^+)$:

$$\begin{aligned} 0 \leq & \int_0^{+\infty}\int_{-1}^{1} \left(|\rho-k|\psi_t + \Phi(t,x,\rho,k)\psi_x\right)\,dx\,dt + \int_{-1}^{1} |\rho_0(x)-k|\psi(0,x)\,dx \\ & + \mathrm{sgn}(k)\int_0^{+\infty} \left(f\left(\rho(t,1-)\right) - f(k)\right)\psi(t,1)\,dt \\ & + \mathrm{sgn}(k)\int_0^{+\infty} \left(f\left(\rho(t,-1+)\right) - f(k)\right)\psi(t,-1)\,dt \\ & + 2\int_0^{+\infty} f(k)\psi\left(t,\xi(t)\right)\,dt. \end{aligned}$$

where $\Phi(t,x,\rho,k) = \mathrm{sgn}(\rho-k)\left(F(t,x,\rho) - F(t,x,k)\right)$

The 1D case: maximum principle

Proposition (ElKhatib-Goatin-Rosini, 2012)

Let $\rho \in \mathbf{C^0}\left(\mathbb{R}^+; \mathrm{BV}(\Omega) \cap \mathbf{L^1}(\Omega)\right)$ be an entropy weak solution. Then

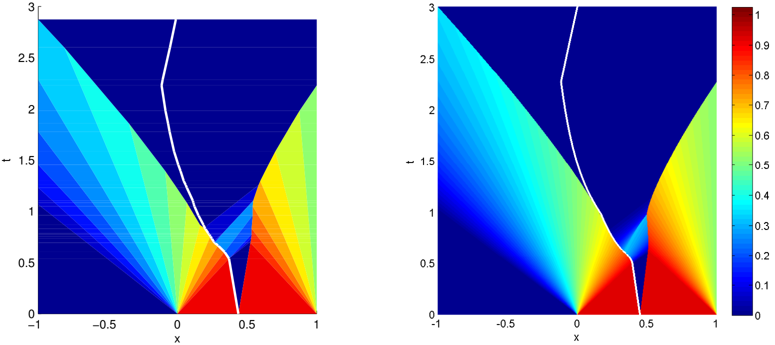
$$0 \leq \rho(t,x) \leq \|\rho_0\|_{\mathbf{L}^\infty(\Omega)}.$$

Characteristic speeds satisfy

$$\begin{aligned} f'\left(\rho^+(t)\right) &\leq \dot{\xi}(t), \text{ if } \rho^-(t) < \rho^+(t), \\ -f'\left(\rho^-(t)\right) &\geq \dot{\xi}(t), \text{ if } \rho^-(t) > \rho^+(t). \end{aligned}$$

The 1D case: wave-front tracking [Goatin-Mimault, SISC 2013]

Riemann-type initial data:

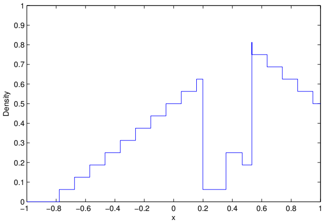


$\Delta\rho = 2^{-4}$ $\Delta\rho = 2^{-10}$

Code freely available at:
`http://www-sop.inria.fr/members/Paola.Goatin/wft.html`

The 1D case: wave-front tracking [Goatin-Mimault, SISC 2013]

Density profile at $t = 0.8$:



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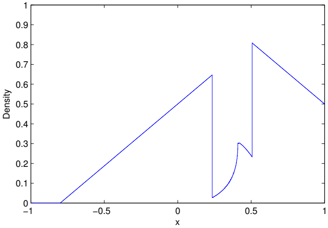


$\Delta\rho = 2^{-10}$

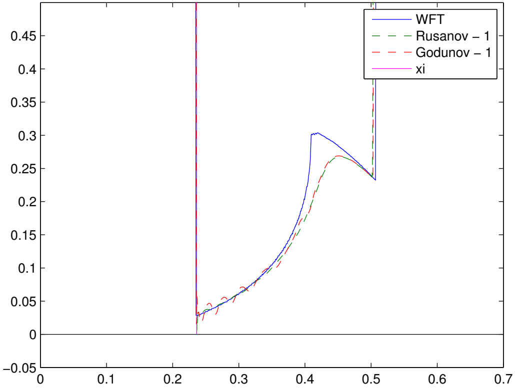
The 1D case: numerical convergence of WFT [Goatin-Mimault, SISC 2013]

ν	$\Delta\rho$	ϵ_ν
5	2^{-5}	$4.280e-2$
6	2^{-6}	$2.164e-2$
7	2^{-7}	$6.141e-3$
8	2^{-8}	$5.048e-3$
9	2^{-9}	$1.755e-3$
10	2^{-10}	$2.091e-3$
11	2^{-11}	$4.305e-4$
12	2^{-12}	$4.347e-4$

Table: $\mathbf{L^1}$-error ϵ_ν for wave-front tracking method between two subsequent discretization meshes $2^{-\nu}$ and $2^{-\nu-1}$. The comparison is done on a cartesian grid with $\Delta x = 10^{-3}$ and $\Delta t = 0.5\Delta x$.

The 1D case: comparison WFT vs FV [Goatin-Mimault, SISC 2013]

Wave-front tracking with $\Delta\rho = 2^{-10}$ and finite volumes with $\Delta x = 1/1500$



The 1D case: comparison WFT vs FV [Goatin-Mimault, SISC 2013]

Δx	Err_G	$\ln(Err_G)/\ln(\Delta x)$	Err_R	$\ln(Err_R)/\ln(\Delta x)$
$1/50$	$7.24e-2$	-0.66	$7.44e-2$	-0.67
$1/100$	$4.56e-2$	-0.66	$4.68e-2$	-0.67
$1/250$	$2.49e-2$	-0.66	$2.55e-2$	-0.67
$1/500$	$1.52e-2$	-0.67	$1.55e-2$	-0.67
$1/1000$	$9.03e-3$	-0.68	$9.12e-2$	-0.68
$1/1500$	$6.66e-3$	-0.69	$6.62e-3$	-0.68

Table: $\mathbf{L^1}$-norm of the error for Godunov and Rusanov schemes compared to wave-front tracking with $\Delta\rho = 2^{-10}$.

Non-local fluxes in 2D

Multi-D integro-differential systems

$$\partial_t U + \mathrm{div}_{\mathbf{x}} F(t, \mathbf{x}, U, U * \eta) = 0$$

with $t \in \mathbb{R}^+$, $\mathbf{x} \in \mathbb{R}^d$, $U(t, \mathbf{x}) \in \mathbb{R}^N$, $\eta(\mathbf{x}) \in \mathbb{R}^{m \times N}$

Theorem [Aggarwal-Colombo-Goatin, SINUM 2015]

For any initial datum $U_o \in (\mathbf{L^1} \cap \mathbf{L}^\infty \cap \mathrm{BV})(\mathbb{R}^2; \mathbb{R}^N_+)$, there exists a solution $U \in \mathbf{C^0}\left(\mathbb{R}_+; \mathbf{L^1}(\mathbb{R}^2; \mathbb{R}^N_+)\right)$. Moreover, for all $k \in \{1, \ldots, N\}$ and for all $t \in \mathbb{R}_+$, the following bounds hold:

$$\begin{aligned}
&\|U(t)\|_{\mathbf{L}^\infty(\mathbb{R}^2;\mathbb{R}^N)} \leq e^{\mathcal{C}\, t(1+\|U_o\|_{\mathbf{L^1}})} \|U_o\|_{\mathbf{L}^\infty(\mathbb{R}^2;\mathbb{R}^N)},\\
&\left\|U^k(t)\right\|_{\mathbf{L^1}(\mathbb{R}^2;\mathbb{R})} = \left\|U^k_o\right\|_{\mathbf{L^1}(\mathbb{R}^2;\mathbb{R})},\\
&\mathrm{TV}(U^k(t)) \leq e^{\mathcal{K}_1 t}\, \mathrm{TV}(U^k_o) + \mathcal{K}_2 \left(e^{\mathcal{K}_1 t} - 1\right),\\
&\|U(t+\tau) - U(t)\|_{\mathbf{L^1}(\mathbb{R}^2;\mathbb{R}^N)} \leq C(t)\, \tau.
\end{aligned}$$

Macroscopic models: summary

Strengths:

- lower computational cost for large crowds
- global description of spatio-temporal evolution
- mathematical tools for well-posedness and numerical approximation
- suitable for posing control and optimization problems

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Thank you for your attention!